Study versus television

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Abstract

The great majority of studies on the effect of school quality on academic outcomes do not take account of changes in student choices concerning time use if school quality, e.g. class size, changes. In particular, students might respond to changes in their school quality by adjusting the time and effort devoted to study (with a consequent change in leisure or market work). In this note we show that such biases could be quite serious and could lead to empirical estimates which either overstate or understate the effects on academic outcomes of changes in school quality. The main parameters governing the sign of the bias are shown to be the extent to which inputs in the education production function are substitutes or complements and how kinked is the benefit from a higher mark. The absolute value of the bias depends also on student ability, the student's distaste for effort and the curvature of the 'production' of the final mark with respect to effort.

Our main conclusion is that reliable estimates of the 'pure' effect of school quality on academic outcomes requires information on time use (and/or academic effort). This suggests a new round of data collection.

1 Introduction

The majority of studies on the effect of 'school quality' on academic outcomes do not take account of changes in student choices concerning time use when school quality changes.[1] In particular, students might respond to changes in their school quality by adjusting the time and effort devoted to study (with a consequent change in leisure or market work). Thus, the 'pure' effects of school quality corresponding to production function parameters may differ from empirical estimates. This is similar to the distinction between production function parameters and average policy effects in Todd and Wolpin (2003) where family inputs are allowed to respond to changes in school quality.

A few papers model and estimate the response of parental inputs to changes in school quality. Houtenville and Conway (2008) consider a theoretical model where student achievement depend on parental effort and school resources, and where parents maximize utility, which is a function of student achievement, leisure and consumption, subject to time and budget constraints. In this model, an increase in school resources may induce parents to increase or reduce their effort depending on the form of utility and production functions. Their empirical analysis indicates that parental effort and per-student spending have positive effects on student achievement, and that some measures of parental effort are affected negatively by per-student spending. However, the estimated effect of per-student spending on achievement is not affected by whether or not parental effort is included in the model. Bonesrønning (2004) finds zero or positive effects on parental effort of reducing class size. Das et al. (2011) consider a dynamic household optimization model where child test scores depend on school and household inputs. Assuming that households make decisions regarding their own inputs before they know the amount of school inputs, they are only able to respond to anticipated changes in school inputs. Using data from Zambia and India the authors find that household school expenditure is reduced when anticipated school grants are increased, and that anticipated grants have no effect on student test scores whereas unanticipated grants have significant positive effects.

To our knowledge, De Fraja et al. (2010) is the only paper which explicitly models and estimates the possible response of student effort to changes in school quality. In their theoretical model student effort, parental effort and school effort are simultaneously determined as a Nash equilibrium. In this very general model, a change in an exogenous variable, e.g. an increase in school resources, may increase or reduce the equilibrium level of effort of students (and parents and schools) depending on the form of the utility and education production functions and the values of exogenous variables. In their empirical analysis,

[1] Studies on the effect of class size include Hanushek (1996), Krueger (1999, 2003), Angrist and Lavy (1999), Case and Deaton (1999), Hoxby (2000), Krueger and Whitmore (2001) and Heinesen (2010). Studies on the effect of teacher quality include Rockoff (2004), Rivkin et al. (2005), Aaronson et al. (2007) and Clotfelter et al. (2007a, 2007b, 2010). Other measures of school quality used in the literature include expenditure per student and the teacher-student ratio (see e.g. the surveys in Hanushek, 1996, Card and Krueger, 1996, and Betts,1996) and the number of teacher hours per student (Browning and Heinesen, 2007).

the measure of student effort is based on (a factor analysis of) general attitude variables such as whether students like school, whether they think homework is boring and whether they want to leave school. The authors do not find any significant effect of class size on their measure of student effort, but they do find that student effort is reduced when 'school effort' is increased, where school effort is based on (a factor analysis of mainly) whether streaming and disciplinary methods are used. They find that student and parental effort are positively correlated (where parental effort is based on a factor analysis of mainly the teacher's opinion of parents' interest in their child's education) and that class size has no effect on parental effort.

The attitudinal variables used by De Fraja et al. (2010) may be poor proxies for effort or time spent on homework, and they may not be expected to be much affected by (marginal) changes in school resources. Furthermore, the authors ignore the important issue of the endogeneity of class size and assume observed class size variation to be exogenous. These problems may explain why they do not find any effect of class size on their measure of student effort.

Some recent papers estimating effects of course-specific (or subject-specific) school quality inputs on student academic outcomes provide indirectly an indication that students' change of effort in response to changes in school quality may be important. Aaronson et al. (2007) estimate effects of teacher quality and find, e.g., statistically significant effects of mathematics teachers on mathematics test scores, but also significant effects of English teachers on mathematics test scores (and of mathematics teachers on English test scores). Heinesen (2010) estimate significant negative effects of subject-specific class size on examination marks in the same subject, but the results also indicate negative effects on marks in other subjects. One interpretation of the effects of subject-specific school inputs on student outcomes in other subjects is that they are due to spill-over effects between subjects induced by student reallocation of effort between subjects.

In this note we consider simple parametric models with endogenous student effort and show that biases due to students' responses to changes in their school quality could be quite serious and could lead to empirical estimates which either overstate or understate the effects on academic outcomes of changes in school quality. The main parameters governing the sign of the bias are shown to be the extent of substitution in the education production function and how kinked is the benefit from a higher mark. The absolute value of the bias also depends on the student's distaste for effort, the curvature of the 'production' of the final mark with respect to effort and the student's ability in the course of study.

Our main conclusion is that reliable estimates of the 'pure' effect of school quality on academic outcomes requires information on time use (and/or academic effort). This suggests a new round of data collection.

In section 2 we discuss models with one course of study. In section 3 we discuss models with more than one course of study. Section 4 contains conclusions.

2 One course of study

2.1 A parametric model

We begin with the simplest case in which a student takes only one course.[2] The outcome is a mark y. This mark is the result of school quality, s, student ability, μ, and student effort, h.[3] To make our main points as cleanly as possible we ignore uncertainty and take a simple parameterisation for the production function:

$$y = s^{\lambda} + \mu h^{\eta} + \varrho s^{\lambda} \mu h^{\eta} \tag{1}$$

Student effort and school quality are normalized so that $0 < h < 1$ and $0 < s \leq 1$. The parameters η and λ capture the curvature in the output with respect to student effort and school quality, respectively. To ensure that production is increasing and concave in both we assume that $0 < \lambda < 1$, $0 < \eta < 1$, $\varrho \geq -1/\overline{\mu}$, and $\varrho > -1$, where $\overline{\mu}$ is the maximum value of μ. If $\varrho < 0$ then s and h are substitutes in production, and if $\varrho > 0$ they are complements. To motivate this function, note that education production may be considered to consist of two learning processes, learning at school (represented by the term s^{λ}) and learning at home doing homework (represented by μh^{η}), and an interaction effect between the two processes represented by the last term in (1). The marginal effect of school resources may be smaller for well-prepared/high-ability students ($\varrho < 0$) if the primary goal of teaching is to ensure that all students obtain a basic level of skills. Also, the marginal effect of effort (and ability) may be smaller when school resources are high: when the learning process at school is more effective there may be smaller returns to effort at home to learn the curriculum. In conventional production functions including the Cobb-Douglas and CES functions, inputs are complements, i.e. the marginal product of one input (e.g. h) increases when the amount of the other input (s) is increased. However, as argued above, h and s may be substitutes in production, so that the marginal product of h is reduced when s increases, and *vice versa*.[4]

In empirical studies a *parameter of interest* is the elasticity of the outcome with respect to school quality:

$$\epsilon = \frac{\partial \ln y}{\partial \ln s} \tag{2}$$

Note that in the parameterisation (1) the elasticity of interest is not independent of effort and student ability. The same is true for the corresponding derivative $\partial y / \partial s$, except when $\varrho = 0$. If effort is fixed then we can recover the parameter of interest from observing variation in y due to experimental variation in s.

[2] In the vastly simplified framework below we consider only a school that has one class. A more general model would distinguish between class quality and school quality and allow for student selection based on within school quality differences.

[3] We use Greek letters to denote preference and prodcution parameters and Latin letters to denote choice variables. Thus s is the choice of the school funding authorities.

[4] Houtenville and Conway (2008) note that school resources and parental inputs may be substitutes in education production.

Typically neither experimental nor non-experimental studies of effects of school quality have access to data on student effort or time use, and therfore the interpretation of estimated effects as education production function parameters presumes that effort is fixed.

However, it is perfectly reasonable to assume that some students might respond to the change in constraints with adjustment in effort expended. To capture this, let preferences be represented by the utility function:

$$u = \frac{y^{1-\sigma}}{1-\sigma} + \delta \ln (1-h) \tag{3}$$

where the maximum time available for study is normalised to unity (as noted above).[5] The parameter δ captures the taste for leisure and the parameter σ (> 0) captures the curvature in the concern about the outcome; higher values of σ give a more kinked benefit function. That is, for higher values of σ the student will have a high return below a threshold value of y ('passing') and a low return above.

2.2 The bias in the empirical elasticity

Denoting the optimal choices by $\left(\hat{h}, \hat{y}\right)$ the *empirical elasticity* is given by:

$$\hat{\epsilon} = \frac{\partial \ln \hat{y}}{\partial \ln s} \tag{4}$$

In general, this will not be equal to the 'true' elasticity ϵ. We define the *empirical bias* as

$$\Delta = \hat{\epsilon} - \epsilon \tag{5}$$

The sign of the bias will depend on the sign of the *effort elasticity*, $\partial \ln \hat{h}/\partial \ln s$. Clearly the bias will be positive ($\hat{\epsilon} > \epsilon$) if the student responds to the increase in school quality by putting in more effort, and the bias will be negative if the effort elasticity is negative. If s and h are substitutes in production ($\varrho \leq 0$) the effort elasticity and bias are negative. But if they are complements ($\varrho > 0$) the marginal product of effort is increased when school quality increases, and therefore it may be optimal for the student to increase effort. Whether it is in fact optimal to increase effort depends on the size of ϱ and the other parameters of the model, especially the curvature of the benefit function with respect to the outcome (σ). Thus, even if s and h are complements it may still be optimal to reduce effort because of the 'income effect': an increase in s enables students to obtain a larger y with less effort (more leisure). We show in the Appendix that the effort elasticity (and therefore the bias) is negative if $\sigma > 1$ or $\varrho \leq 0$. We now examine further the determinants of the direction and size of the bias.

Although simple, this parameterisation does not yield closed form expressions for the elasticity of interest. We therefore have to resort to simulations to

[5] It is straightforward to allow for alternative uses of time such as market work. This complicates the notation and analysis without adding much of significance to the main points.

illustrate how the empirical elasticity varies with the parameters. Without loss of generality we can take $\lambda = 0.4$.[6]

We take a grid over the values given in table 1 (and calculate the elasticity at $s = 1$).[7] These values are, of course, wholly arbitrary and serve only to illustrate the variation in the bias of the empirical elasticity. Since the maximum value of μ is here equal to 10, concavity is ensured when $\varrho \geq -0.1$.

Parameter	**minimum**	**maximum**	**grid step**
μ	1	10	1
δ	0.5	2.0	0.1
σ	0.45	1.95	0.1
η	0.1	0.6	0.1
ϱ	-0.075	0.075	0.025

Table 1: Simulation parameter values

With this range of parameter values the minimum and maximum bias of the empirical elasticity are -0.12 and 0.00, respectively. Thus, even when ϱ attains its maximum value of the grid (0.075) the bias is not positive for any combination of the other parameters within the grid of table 1. Table 2 shows that the parameter values that induce the extreme values of the bias are very different. The table also shows the true and empirical elasticities. The true elasticity is calculated holding h fixed at the optimal level given the parameters, whereas h is adjusted to its new optimal level when calculating the empirical elasticity. As expected, the largest negative bias is obtained when s and h are strong subsitutes in production (ϱ attains its minimum). Also, the bias is more negative if the benefit function is more curved (high σ), if the student has a higher taste for leisure (high δ), if the elasticity of output with respect to effort is high (high η), and/or if student ability (μ) is relatively low (although in this case not at its minimum).

True	Empirical	Bias	Parameters				
elasticity	elasticity		$\boldsymbol{\mu}$	$\boldsymbol{\delta}$	$\boldsymbol{\sigma}$	$\boldsymbol{\eta}$	$\boldsymbol{\varrho}$
0.249	0.125	-0.124	3	2.0	1.95	0.6	-0.075
0.062	0.062	-0.000	10	0.5	0.45	0.1	0.075

Table 2: Extreme values of the bias of the empirical elasticity

When s and h are strong complements in production the bias of the empirical elasticity may be substantially positive. This is illustrated in table 3 where ϱ is fixed at 2. Here the largest negative bias is obtained for about the same values of $(\mu, \delta, \sigma, \eta)$ as in table 2 (except that μ is 2 instead of 3), whereas the largest positive bias (of 0.055) is obtained for the same parameter values except

[6] Results are qualitatively the same for other values of λ.
[7] Choosing other values of s produces qualitatively similar results.

that σ (the curvature of the benefit function) is at its minimum instead of its maximum.

True elasticity	Empirical elasticity	Bias	Parameters $\boldsymbol{\mu}$	$\boldsymbol{\delta}$	$\boldsymbol{\sigma}$	$\boldsymbol{\eta}$	$\boldsymbol{\varrho}$
0.326	0.217	-0.109	2	2.0	1.95	0.6	2
0.299	0.354	0.055	2	2.0	0.45	0.6	2

Table 3: Extreme values of the bias of the empirical elasticity when school quality and effort are strong complements (rho=2)

Within any school, we would expect that the parameters $(\mu, \delta, \sigma, \eta, \varrho)$ are heterogeneous. For example, how important it is to attain more than a simple 'passing' grade will vary from student to student, implying heterogeneity in the parameter σ. Moreover, the distributions of these parameters may not be independent. For example, high ability students (high μ) who aspire to further education may have a lower concern for simply passing.

2.3 Differential effects

The results of Summers and Wolfe (1977), Krueger (1999), Angrist and Lavy (1999), Browning and Heinesen (2007) and Heinesen (2010) indicate that reducing class size has larger positive effects for students from disadvantaged backgrounds, and Heinesen (2010) also finds that low-ability students benefit significantly more than high-ability students. Aaronson et al. (2007) find that teacher-quality effects are relatively larger for lower-ability students.

The simple model described above is consistent with these findings since the empirical elasticity, $\hat{\epsilon}$, is decreasing in student ability, μ. The absolute value of the empirical effect of school resources on marks $(\partial\hat{y}/\partial s)$ is also decreasing in μ for many combinations of values for the other parameters. When $\varrho = 0$ (i.e., s and h are neither substitutes nor complements in production), the derivative $\partial y/\partial s$ in the production function (1) holding h fixed does not depend on μ. However, the empirical effect $\partial\hat{y}/\partial s$ depends on μ because students respond to changes in school quality by changing effort. This is illustrated in figure 1 which shows how $\hat{\epsilon}$ and $\partial\hat{y}/\partial s$ vary with μ in the model consisting of (3) and (1) where $(\delta, \sigma, \eta) = (1.0, 1.05, 0.4)$. In the lower part of the figure $\varrho = 0$, and the lower right panel shows that $\partial\hat{y}/\partial s$ is decreasing in μ. The lower left panel shows that the empirical elasiticity decreases much more in μ. This is not surprising since y determined by the production function (1) holding h fixed and also the optimal value $\hat{y}$ are strongly increasing in μ. In the upper part of the figure $\varrho = -0.05$ (i.e., s and h are substitutes in production). Whereas the empirical elasticity varies with μ in much the same way in the two parts of the figure, $\partial\hat{y}/\partial s$ decreases much more in the upper part of the figure, where s and h are substitutes, than in the lower part.

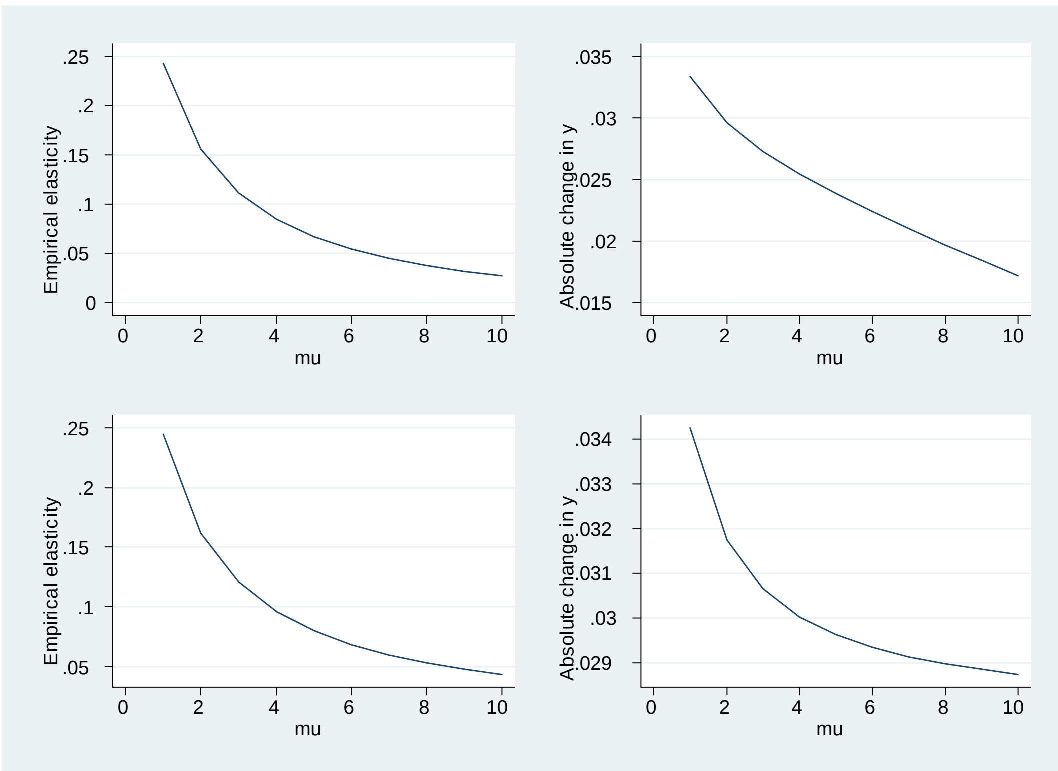

Figure 1: Empirical elasticity and absolute change in outcome when school resources are increased by 10%. The two upper panels are for rho=-0.05, the two lower panels are for rho=0.

2.4 A pass/fail mark

If in this simple model the curvature parameter in the utility function is above a threshold ($\sigma > 1$), students respond to an increase in school quality by *decreasing* their effort. Conversely, if $\sigma < 1$ (and $\varrho > 0$), students may in some cases respond by increasing their effort. To investigate in more detail the effect of the curvature of the benefit function we consider an extreme case in which:

$$u = \mathbf{I}\left(y \geq y^*\right) + \delta \ln\left(1 - h\right) \tag{6}$$

where $\mathbf{I}\left(y \geq y^*\right)$ is an indicator function that takes value unity if $y \geq y^*$ and zero otherwise (with $y^* > s^\lambda$). This corresponds to a pass/fail mark. In this case students will either set:

$$\begin{aligned} &h = 0 \Rightarrow u = 0 \\ &h = \left(\frac{y^* - s^\lambda}{\mu(1 + \varrho s^\lambda)}\right)^{\frac{1}{\eta}} \Rightarrow u = 1 + \delta \ln\left(1 - \left(\frac{y^* - s^\lambda}{\mu(1 + \varrho s^\lambda)}\right)^{\frac{1}{\eta}}\right) \end{aligned} \tag{7}$$

where we assume that the passing grade is attainable for some feasible level of effort: $y^* < s^\lambda + \mu(1 + \varrho s^\lambda)$. The student chooses to exert effort if:

$$\delta \leq -\left(\ln\left(1 - \left(\frac{y^* - s^\lambda}{\mu(1 + \varrho s^\lambda)}\right)^{\frac{1}{\eta}}\right)\right)^{-1} \tag{8}$$

This illustrates that, *ceteris paribus*, a pass grade is more likely if complementarity in production, student ability or school quality are high or if the student has a low taste for leisure.[8] For a given level of school quality we have three groups of students: bad fails; marginal fails (students who failed but were close to choosing to pass) and passes. If we increase school quality then the bad fails continue to exert no effort and fail, the marginal fails increase their effort (the level given in (7) with the new level of s) and pass students reduce their effort and still pass. Thus we have three different responses to the policy change: negative, zero and positive.

3 More than one course of study

When there are more than one course of study, there may be spill-over effects between courses or subjects in the sense that subject-specific school inputs may affect student outcomes in other subjects through student reallocation of effort between subjects. This may be illustrated by an extension of the above model framework to the more general case with more than one course of study. For

[8] It is easily shown that $\frac{\partial}{\partial s}\left(\frac{y^* - s^\lambda}{\mu(1+\varrho s^\lambda)}\right) < 0 \Leftrightarrow 1 + \varrho y^* > 0$, and that this inequality holds because of the restriction $\varrho \geq -1/\overline{\mu}$.

simplicity, consider the case with two courses. We assume that the utility function is additive in the two outcomes (with the same curvature parameter) and leisure:

$$v = \frac{y_1^{1-\sigma}}{1-\sigma} + \frac{y_2^{1-\sigma}}{1-\sigma} + \delta \ln(1 - h_1 - h_2) \tag{9}$$

where y_i and h_i are the outcome (a mark) and student effort in subject i, respectively.

We allow student ability (μ_i) to differ between subjects, but for simplicity we assume that the other parameters in the two subject-specific production functions are identical, and we assume their form to be similar to (1):

$$y_i = s_i^\lambda + \mu_i h_i^\eta + \varrho s_i^\lambda \mu_i h_i^\eta, \qquad i = 1, 2 \tag{10}$$

Denoting the optimal choices by $(\hat{h}_1, \hat{h}_2, \hat{y}_1, \hat{y}_2)$ we may consider four empirical elasticities, namely elasticities of the two outcomes with respect to each of the two school quality inputs:

$$\hat{\epsilon}_{ii} = \frac{\partial \ln \hat{y}_i}{\partial \ln s_i}, \qquad i = 1, 2 \tag{11}$$

$$\hat{\epsilon}_{ij} = \frac{\partial \ln \hat{y}_i}{\partial \ln s_j}, \qquad i, j = 1, 2, \quad i \neq j \tag{12}$$

The two empirical 'own resource' elasticities in (11) consist of a direct effect of increased school resources in subject i on academic outcome in the same subject and an indirect effect through changed effort in subject i. The two empirical cross-elasticities (12) are different from zero if an increase in school resources in one subject induces students to change effort in the other subject. The true own-resource elasticities, $\epsilon_{ii} = \partial \ln y_i / \partial \ln s_i$, consist of only the direct effect, holding effort fixed. The true cross-elasticities, $\epsilon_{ij} = \partial \ln y_i / \partial \ln s_j, i \neq j$, are zero. The sign of the bias of each of the four empirical elasticities $\hat{\epsilon}_{ij}$ $(i, j = 1, 2)$ is equal to the sign of the corresponding effort elasticity $\partial \ln \hat{h}_i / \partial \ln s_j$. We show in the Appendix that $\partial \hat{h}_i / \partial s_i < 0$ and $\partial \hat{h}_i / \partial s_j > 0$ $(i, j = 1, 2;\ i \neq j)$ if $\sigma > 1$ or $\varrho \leq 0$.

If we take a grid over the same values of the parameters as given in table 1, where now both μ_1 and μ_2 vary between 1 and 10, we obtain the extreme values of the bias of the elasticity $\hat{\epsilon}_{11}$ and the extreme value of (the bias of) the cross-elasticity $\hat{\epsilon}_{12}$ and the associated parameters shown in table 4.[9] The bias of the empirical 'own resource' elasticity $\hat{\epsilon}_{11}$ has extreme values -0.12 and -0.00, and the same is true for the bias of $\hat{\epsilon}_{22}$ (not shown in the table since the model is symmetric in the two subjects). The bias of $\hat{\epsilon}_{11}$ (and $\hat{\epsilon}_{22}$) vary with the parameters in basically the same way as the bias of $\hat{\epsilon}$ in the one-course case (except for the dependence on ability in the other subject): The extreme negative bias is obtained when σ, δ and η are at their maximum, when ϱ is at its minimum, and when ability in the two subjects is low (although not at the minimum). The maximum bias (zero) is obtained when when σ, δ and η are at

[9] The elasticities are calculated at $\lambda = 0.4$ and $s_1 = s_2 = 0.5$.

their minimum, when ϱ is at its maximum, and when ability is at its maximum in the same subject and at its minimum in the other subject. The two cross elasticities $\hat{\epsilon}_{12}$ and $\hat{\epsilon}_{21}$ have extreme values -0.00 and 0.05. The extreme positive value of $\hat{\epsilon}_{12}$ is obtained when σ, η and μ_1 are at their maximum values, and when ϱ, δ and μ_2 are at their minimum values. Thus, the elasticity of outcome in one subject with respect to school resources in the other subject is high when ϱ is negative and numerically large, σ is high, ability in the first subject is high and ability in the other subject is low. The minimum of $\hat{\epsilon}_{12}$ is obtained when $\varrho = 0$ and when σ, δ, μ_1 and μ_2 are high, and η is low.

	True elasticity	Empirical elasticity	Bias	Parameters					
				$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$	$\boldsymbol{\delta}$	$\boldsymbol{\sigma}$	$\boldsymbol{\eta}$	$\boldsymbol{\varrho}$
$\hat{\epsilon}_{11}$	0.253	0.131	-0.121	2	2	2.0	1.95	0.6	-0.075
	0.049	0.049	-0.000	10	1	0.5	0.45	0.1	0.075
$\hat{\epsilon}_{12}$	0.0	-0.001	-0.001	9	10	2.0	1.95	0.1	0.000
	0.0	0.052	0.052	10	1	0.5	1.95	0.6	-0.075

Table 4: Extreme biases of empirical elasticities in model with two courses of study

Table 5 illustrates that when school quality and student effort are strong complements in production ($\varrho = 2$ in the table) then the bias of $\hat{\epsilon}_{11}$ may be substantially positive (as for $\hat{\epsilon}$ in the model with a single subject) and $\hat{\epsilon}_{12}$ may be substantially negative. The maximum bias of $\hat{\epsilon}_{11}$ is obtained when σ and δ are small and η and μ_1 and μ_2 are high. The minimum of $\hat{\epsilon}_{12}$ is obtained for the same parameter values, except that μ_1 is small. However, table 5 also shows that when σ is large, the bias of $\hat{\epsilon}_{11}$ may be substantially negative and $\hat{\epsilon}_{12}$ may be substantially positive even when s and h are strong complements in production.

	True elasticity	Empirical elasticity	Bias	Parameters					
				$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$	$\boldsymbol{\delta}$	$\boldsymbol{\sigma}$	$\boldsymbol{\eta}$	$\boldsymbol{\varrho}$
$\hat{\epsilon}_{11}$	0.237	0.169	-0.068	10	10	0.5	1.95	0.6	2
	0.210	0.308	0.098	10	10	0.5	0.45	0.6	2
$\hat{\epsilon}_{12}$	0.0	-0.069	-0.069	3	10	0.5	0.45	0.6	2
	0.0	0.032	0.032	10	1	0.5	1.85	0.6	2

Table 5: Extreme biases of empirical elasticities in model with two courses of study when school quality and effort are strong complements (rho=2)

Figure 2 shows how the empirical elasticities $\hat{\epsilon}_{ij}$ and the absolute change in the outcomes $\mathrm{D}\hat{y}_{ij}$, which is the increase in $\hat{y}_i$ when s_j is increased by 10%, vary with ability in subject 1 when $(\mu_2, \delta, \sigma, \eta, \varrho) = (5, 0.5, 1.95, 0.6, -0.05)$. Thus, the values of δ, σ and η are chosen so that the cross elasticities are relatively large. The own-subject elasticity $\hat{\epsilon}_{11}$ and the absolute change $\mathrm{D}\hat{y}_{11}$ are decreasing in μ_1 corresponding to the results in the one-subject case. The

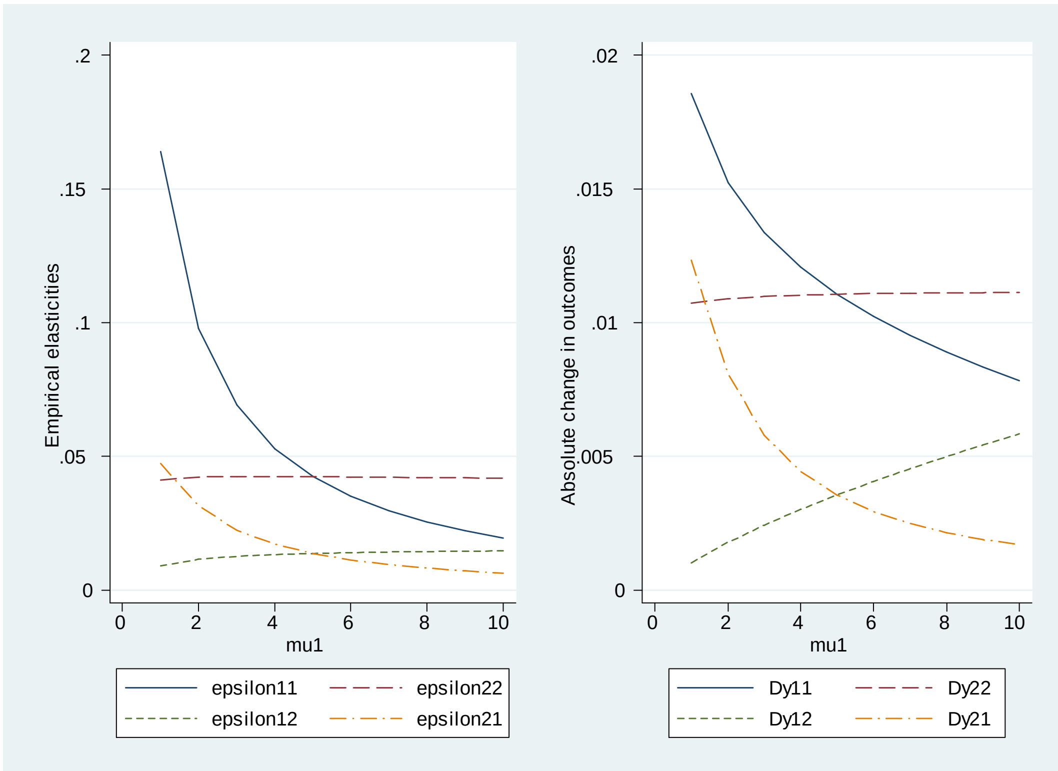

Figure 2: Empirical elasticities and absolute changes in outcomes when school resources are increased by 10%

other own-subject elasticity $\hat{\epsilon}_{22}$ (and $\mathrm{D}\hat{y}_{22}$) do not depend much on μ_1, while the cross elasticity $\hat{\epsilon}_{21}$ (and $\mathrm{D}\hat{y}_{21}$) are decreasing in μ_1, and $\hat{\epsilon}_{12}$ (and $\mathrm{D}\hat{y}_{12}$) are increasing in μ_1. The figure illustrates that the cross elasticities may be rather large compared to the own-subject elasticities. For instance, when $\mu_1 = 5$ the cross elasticities are about one third of the own-subject elasticities. Thus, when school resources in one course is changed, this may have substantial effects on outcomes also in other courses, and the mechanism behind these cross effects in this simple model is students' reallocation of effort between subjects: When school resources in one course increase it may be optimal for students to reduce effort in this course and increase leisure and effort in the other course. In figure 2 $\varrho = -0.05$ so that s_i and h_i are assumed to be substitutes in production. Setting ϱ equal to zero produces a rather similar figure although the cross elasticities are a little smaller compared to the own-course elasticities (for $\mu_1 = 5$ the ratio is 0.27). Assuming inputs to be strong complements in production implies that the ratios of cross elasticities to own-course elasticities are smaller.

4 Conclusion

Typically, studies on the effect of school quality on academic outcomes do not take account of students' responses regarding academic effort or time use. Applying simple parametric models we have shown that students' effort or time-use responses to changes in school quality may cause large differences between the empirical elasticity of changes in school quality and the education production function elasticity (holding student effort constant). The main parameters determining the sign of the bias are the extent of substitution between effort and school quality in the production function and how kinked is the benefit from a higher mark in the student utility function. If effort and school quality are substitutes in production and/or if there is a marked kink in the benefit function, students will tend to reduce effort when school quality is increased implying a negative bias in the empirical elasticity. The value of the bias also depends on the student's distaste for effort, the curvature of the production function with respect to effort and the student's ability in the course of study. In models with two courses of study we show that an increase in school resources in one course may reduce effort in that course, implying a negative bias of the empirical 'own-resource elasticity', but increase effort in the other course implying a positive bias in the empirical 'cross elasticity'.

Our main conclusion is that reliable estimates of the 'pure' effect of school quality on academic outcomes require - in addition to exogenous variation in school quality - information on time use and/or academic effort. This suggests a new round of data collection.

5 Appendix

5.1 Effect of school quality on effort in the model with one course of study

We show that $\partial\hat{h}/\partial s < 0$ if $\sigma > 1$ or $\varrho \leq 0$. The marginal effect of school quality on effort is found by inserting the production function (1) into the utility function (3) and differentiating the first-order condition:

$$\begin{aligned} \frac{\partial\hat{h}}{\partial s} &= -\frac{\partial^2 u/\partial\hat{h}\partial s}{\partial^2 u/\partial\hat{h}^2} \\ \frac{\partial^2 u}{\partial\hat{h}\partial s} &= \lambda s^{\lambda-1}\mu\eta h^{\eta-1}y^{-\sigma}[\varrho(1-\sigma)-\sigma y^{-1}] \\ \frac{\partial^2 u}{\partial\hat{h}^2} &= \mu\eta h^{\eta-2}(1+\varrho s^{\lambda})y^{-\sigma}[(\eta-1)-\sigma y^{-1}\mu\eta h^{\eta}(1+\varrho s^{\lambda})] - \frac{\delta}{(1-h)^2} < 0 \end{aligned} \tag{13}$$

Thus, the sign of $\partial\hat{h}/\partial s$ is equal to the sign of $\partial^2 u/\partial\hat{h}\partial s$. It is obvious that

$\partial\hat{h}/\partial s < 0$ if ($\varrho \leq 0$ and $\sigma < 1$) or if ($\varrho \geq 0$ and $\sigma > 1$). However, $\partial\hat{h}/\partial s$ is also negative when $\varrho < 0$ and $\sigma > 1$. Thus, when $\varrho < 0$ and $\sigma > 1$ we have

$$\begin{aligned} \frac{\partial^2 u}{\partial\hat{h}\partial s} < 0 &\Leftrightarrow \varrho(1-\sigma) - \sigma(s^\lambda + \mu h^\eta + \varrho s^\lambda \mu h^\eta)^{-1} < 0 \\ &\Leftrightarrow s^\lambda + \mu h^\eta + \varrho s^\lambda \mu h^\eta < \frac{\sigma}{1-\sigma}\frac{1}{\varrho} \end{aligned} \tag{14}$$

This inequality always holds given the assumed restrictions on the parameters. The RHS of the inequality tends towards its lower limit (10) from above when $\sigma \to \infty$ and $\varrho = -0.1$ (which is the minimum of ϱ). For $\varrho = -0.1$ the LHS is always below 10 (since the maximum of s and μ are 1 and 10, respectively, and $h < 1$). It is easily seen that the inequality also holds when ϱ is increased above -0.1. Thus, we have shown that $\partial\hat{h}/\partial s < 0$ if $\varrho \leq 0$ or $\sigma > 1$.

5.2 Effect of school quality on effort in the model with two courses of study

We show that $\partial\hat{h}_i/\partial s_i < 0$ and $\partial\hat{h}_i/\partial s_j > 0$ $(i,j = 1,2,\ i \neq j)$ if $\sigma > 1$ or $\varrho \leq 0$. Inserting the production functions (10) into the utility function (9) and differentiating the first-order conditions, we have:

$$\begin{aligned} \frac{\partial\hat{h}_i}{\partial s_i} &= -\frac{\partial^2 v}{\partial\hat{h}_i\partial s_i}\frac{\partial^2 v}{\partial\hat{h}_j^2}/D \\ \frac{\partial\hat{h}_i}{\partial s_j} &= \frac{\partial^2 v}{\partial\hat{h}_j\partial s_j}\frac{\partial^2 v}{\partial\hat{h}_i\partial\hat{h}_j}/D \\ D &= \frac{\partial^2 v}{\partial\hat{h}_1^2}\frac{\partial^2 v}{\partial\hat{h}_2^2} - \left(\frac{\partial^2 v}{\partial\hat{h}_1\partial\hat{h}_2}\right)^2 \end{aligned} \tag{15}$$

where

$$\begin{aligned} \frac{\partial^2 v}{\partial\hat{h}_i\partial s_i} &= \lambda s_i^{\lambda-1}\mu_i\eta h_i^{\eta-1}y_i^{-\sigma}[\varrho(1-\sigma) - \sigma y_i^{-1}] \\ \frac{\partial^2 v}{\partial\hat{h}_i^2} &= A_i - \frac{\delta}{(1-h_1-h_2)^2} \quad , \qquad \frac{\partial^2 v}{\partial\hat{h}_1\partial\hat{h}_2} = -\frac{\delta}{(1-h_1-h_2)^2} \\ A_i &= \mu_i\eta h_i^{\eta-2}(1+\varrho s_i^\lambda)y_i^{-\sigma}[(\eta-1) - \sigma y_i^{-1}\mu_i\eta h_i^\eta(1+\varrho s_i^\lambda)] \end{aligned} \tag{16}$$

Since $A_i < 0$, we have $\partial^2 v/\partial\hat{h}_i^2 < 0$. Furthermore, $D = A_1A_2 - (A_1+A_2)\delta/(1-h_1-h_2)^2 > 0$. Thus, $\partial\hat{h}_i/\partial s_i < 0$ and $\partial\hat{h}_i/\partial s_j > 0$ iff $\varrho(1-\sigma) - \sigma y_i^{-1} < 0$, and this inequality holds if $\sigma > 1$ or $\varrho \leq 0$ by arguments similar to the one-course case.

6 References

Aaronson, D.; L. Barrow & W. Sander (2007): Teachers and Student Achievement in the Chicago Public High Schools. *Journal of Labor Economics*, 25(1): 95-135.

Angrist, J.D. & V. Lavy (1999): Using Maimonides' rule to estimate the effect of class size on scholastic achievement. *Quarterly Journal of Economics*, 114(2): 533-575.

Betts, J.R. (1996): Is there a link between school inputs and earnings? Fresh scrutiny of an old literature. In G. Burtless: *Does Money Matter? The Effect of School Resources on Student Achievement and Adult Success*, pp. 141–191, Washington DC: Brookings Institution.

Bonesrønning, H. (2004): The determinants of parental effort in education production: Do parents respond to changes in class size? *Economics of Education Review* 23: 1-9.

Browning, M. & E. Heinesen (2007). Class size, teacher hours and educational attainment. *Scandinavian Journal of Economics*, 109(2): 415-438.

Card, D. & A.B. Krueger (1996): Labor market effects of school quality: Theory and evidence. In G. Burtless: *Does Money Matter? The Effect of School Resources on Student Achievement and Adult Success*, pp. 97–140, Washington DC: Brookings Institution.

Case, A. & A. Deaton (1999): School inputs and educational outcomes in South Africa. *Quarterly Journal of Economics*, 114(3): 1047-1084.

Clotfelter, C.T.; H.F. Ladd & J.L. Vigdor (2007a): How and why do teacher credentials matter for student achievement? NBER Working Paper 12828.

Clotfelter, C.T.; H.F. Ladd & J.L. Vigdor (2007b): Teacher credentials and student achievement: Longitudinal analysis with student fixed effects. *Economics of Education Review*, 26: 673-682.

Clotfelter, C.T.; H.F. Ladd & J.L. Vigdor (2010): Teacher credentials and student achievement in high school: a cross-subject analysis with student fixed effects. *Journal of Human Resources*, 45(3): 655-681.

Das, J.; S. Dercon, J. Habyarimana, P. Krishnan, K. Muralidharan & V. Sundararaman (2011): School inputs, household substitution, and test scores. NBER Working Paper 16830.

De Fraja, G.; T. Oliveira & L. Zanchi (2010): Must try harder: Evaluating the role of effort in educational attainment. Review of Economics and Statistics 92(3): 577-597.

Dustmann, C., N. Rajah & A. van Soest (2003): Class size, education, and wages. *Economic Journal*, 113: F99-F120.

Hanushek, E.A. (1996): School resources and student performance. In G. Burtless (ed.): *Does money matter? The effect of school resources on student achievement and adult success*, pp. 43-73. Washington, D.C: Brookings Institution Press.

Heinesen, E. (2010): Estimating class-size effects using within-school variation in subject-specific classes. *Economic Journal*, 120: 737-760.

Houtenville, A.J. & K.S. Conway (2008): Parental effort, school resources, and student achievement. *Journal of Human Resources*, 43(2): 437-453.

Hoxby, C. (2000): The effects of class size on student achievement: New evidence from population variation. *Quarterly Journal of Economics*, 115: 1239-1285

Krueger, A.B. (1999): Experimental estimates of educational production functions. *Quarterly Journal of Economics*, 114(2): 497-532.

Krueger, A.B. (2003): Economic considerations and class size. *Economic Journal*, 113: F34-F63.

Krueger, A.B. & D.M. Whitmore (2001): The effect of attending a small class in the early grades on college-test taking and middle school test results: Evidence from project STAR. *Economic Journal*, 111: 1-28.

Lazear, E.P. (2001) Educational production. *Quarterly Journal of Economics*, 116(3), pp. 777-803.

Rivkin, S.; E.A. Hanushek & J.F. Kain (2005): Teachers, schools, and academic achievement. *Econometrica*, 73(2): 417-457.

Rockoff, J.E. (2004): The Impact of Individual Teachers on Student Achievement: Evidence from Panel Data. *American Economic Review*, 94(2): 247-252.

Summers, A.A., & B.L. Wolfe (1977): Do schools make a difference? *American Economic Review*, 67(4): 639-652.

Todd, P.E. & K.I. Wolpin (2003): On the specification and estimation of the production function for cognitive achievement. *Economic Journal*, 113: F3-F33.